

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 214401

Roll No.

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**MCA.**

**(SEM. IV) THEORY EXAMINATION 2013-14**  
**DESIGN AND ANALYSIS OF ALGORITHMS**

*Time : 3 Hours**Total Marks : 100***Note :-** Attempt questions from each Section as indicated.**SECTION-A**

1. Attempt **all** parts of the section. **(2×10=20)**

- (a) Define asymptotic notation  $O$ ,  $\Omega$  and  $\theta$ .
- (b) Show that for any constant  $a$  and  $b$ , where  $b > 0$ ,  
 $(n + a)^b = \theta(n^b)$ .
- (c) Use the master method to give tight asymptotic bounds for the recurrence

$$T(n) = 3T\left(\frac{n}{4}\right) + n \log_2^n.$$

- (d) Show that if  $n \geq 1$ , then for any  $n$ -key B-tree 'T' of height  $h$  and minimum degree  $t \geq 2$ ,  $h \leq \log_t \frac{n+1}{2}$ .
- (e) What do you mean by randomized algorithm ? Give an example.
- (f) What is minimum-weight spanning tree ?

- (g) What do you mean by dynamic programming ? How is it different from divide and conquer method.
- (h) Find all solutions to the following equation for x,  
 $35x \equiv 10 \pmod{50}$ .
- (i) Define NP and NP-complete class of problem.
- (j) Show that Bitonic-sorter[n], where n is an exact power of 2, contains  $\theta(n \log_2^n)$  comparators.

### SECTION-B

**Note :-** Attempt any **three** questions out of **five** questions from the following. **(3×10=30)**

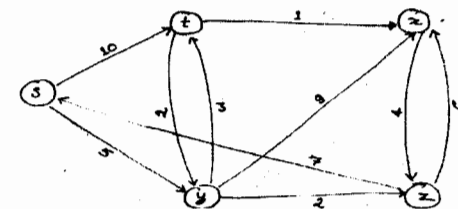
2. (a) Illustrate the operation of merge-sort on the array  $A = \langle 5, 2, 4, 7, 1, 3, 2, 6 \rangle$ . Prove that running of merge sort is  $\theta(n \log_2^n)$ .
- (b) Write the Algorithm for counting sort.
3. (a) Explain the properties of binomial tree.
- (b) Why don't we allow a minimum degree of  $t = 1$  for B-tree.
4. (a) Write a procedure to Huffman code.
- (b) Determine the LCS of  $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$  and  $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$ .
5. (a) What is the running time of BFS if its input graph is represented by an adjacency matrix and the algorithm is modified to handle this form of input ?
- (b) Discuss the procedure of Bellman-Ford Algorithm.
6. (a) Working modulo  $q = 11$ , how many spurious hits does the rabin-karp matcher encounter in the text  $T = 3141592653589793$  when looking for the pattern  $P = 26$  ?

- (b) Show that Hamiltonian-path problem is NP-Complete.

### SECTION-C

**Note :-** Attempt any **one** part from each question. All questions are compulsory. **(5×10=50)**

7. (a) Write an algorithm to sort the given array of elements using quick sort, illustrate the operation of quick sort on the array  $A = \langle 5, 13, 2, 25, 7, 17, 20, 8 \rangle$  analyze its running time in best case also.
- (b) With the help of recursion tree, provide tight asymptotic bound on the solution of:  
 $T(n) = T(n/2) + T(n/4) + T(n/8) + n$ .
8. (a) Explain Prim's algorithm using suitable example.
- (b) Explain Dijkstra's algorithm. Illustrate the operation of Dijkstra algorithm on the following graph ('s' is the source vertex).



9. (a) Show the results of inserting the key  
 F, S, Q, K, C, L, H, T, V, W, M, R, N, P, A, B, X, Y, D, Z, E  
 in order into an empty B tree with degree 4. Only draw the configurations of the tree just before nodes must split, and also draw the final configuration.
- (b) Write Pseudocode for Binomial Heap Union ( $H_1, H_2$ ).

10. (a) Discuss dynamic programming. Write the procedure for matrix chain multiplication.
- (b) Define a knapsack problem and describe its formation. Find the optimal solution to the knapsack instance  $n = 5$ ,  $W = [20, 30, 40, 10, 7]$ ,  $P = [7, 8, 9, 1, 6]$  and  $C = 80$  using Greedy method.
11. (a) Use Strassen's algorithm to compute the product of two given square matrices :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} 10 & 11 \\ 12 & 13 \end{bmatrix}$$

Also compute the number of multiplication and addition/subtraction operations in the process.

- (b) Discuss RSA public-key-cryptography algorithm.